# Cut-off and Phase Constants of Partially Filled Axially Magnetized, Gyromagnetic Waveguides Using Finite Elements

Bernice M. Dillon, Andrew A. P. Gibson, and Jon P. Webb

*Abstract*—**A three component vector finite element formulation to model the propagation characteristics of partially filled, axially magnetized, gyromagnetic waveguides is described. Covariant-projection elements have been used to avoid spurious modes and periodic boundary conditions have been implemented to improve numerical efficiency. The classic quadratic functional derived from the vector, curlcurl, magnetic field equation is suited to evaluating the cut-off planes of gyrotropic waveguides. A known field transformation is used to reformulate the functional into a form convenient for calculating phase constants. Cut-off and phase constant solutions are presented for both fully and partially filled, longitudinally magnetized, ferrite loaded waveguides.**

## I. Introduction

An UNDERSTANDING of cut-off conditions, phase constants and mode nomenclature are necessary in waveguide design problems. Boundary value problems associated with gyromagnetic waveguides often produce intractable characteristic equations describing complex modal behaviour [1]–[4]. An alternative approach is to formulate such problems in terms of a finite element procedure [5]–[7]. Inhomogeneous gyromagnetic waveguide cross sections are solved here using the finite element method with covariant-projection elements to eliminate spurious modes [8], [14]. The particular case treated is longitudinally magnetized ferrite structures. This class of boundary problem is of interest in the area of ferrite phase shifters, resonators, circulators and tunable filters [9].

The finite element analysis of a vector field problem is usually based on either an axial $(E_z, H_z)$ formulation or on a three component ($\overline{E}$ or $\overline{H}$) field formulation. Gibson and Helszajn [6] have used an axial component formulation to study the characteristics of ferrite filled elliptical waveguides. One difficulty with axial component formulations is that for inhomogeneous geometries it is not easy to impose the boundary conditions between media interfaces [10]. The three component field formulation requires no modification for media interfaces and so can be applied to arbitrary inhomogeneous geometries. Such a formulation was first proposed by Konrad [5] and since then it has been widely used. Its eigensolution provides the three component vector field as the eigenvector for each modal eigenvalue frequency. Recently, this formulation has been used with edge elements to avoid spurious modes in the study of ferrite cavities [11]. By setting the phase constant to zero the cut-off planes of gyromagnetic structures are readily evaluated. Cut-off curves are presented for the first few modes of longitudinally magnetized, circular waveguides using this approach. Both fully and partially filled cross sections are examined. For phase constant evaluation however this approach is inefficient as the phase constant is a prerequisite to the analysis.

A direct method of calculating phase constants from this functional has been described in the literature [12]. It requires the fields to be transformed such that the phase constant squared becomes the eigenvalue. The Hermitian tensors of the medium must be reducible into transverse and axial components. Longitudinally magnetized structures satisfy this prerequisite. Spurious mode free eigensolutions of phase constants and fields are obtained by using this transformed functional with covariant-projection elements. Finite element phase constant calculations are made for both partially and completely filled longitudinally magnetized circular ferrite waveguides. All finite element calculations are in good agreement with previously published results [2].

The circular geometries treated here are modelled using covariant-projection elements. These are rectangular elements with curvilinear sides, which have a special function space designed to avoid spurious modes [8], [14]. The shape functions used within these elements are defined so that the irrotational solutions are modelled correctly. Numerical efficiency has been improved for the axisymmetric geometries treated by introducing periodic boundary conditions and discretizing one half of the geometry only.

## II. Formulation for the Cut-Off Plane

In longitudinally-magnetized ferrite structures, the static magnetic field is applied in the direction of electromagnetic wave propagation. In the following analysis this coincides with the $z$-axis of the Cartesian co-ordinate system. The propagation characteristics of the waveguide are dependent on the applied magnetic field; for example, electromagnetic waves with clockwise and anti-clockwise circular-polarization display different phase velocities along the waveguide. Hence the ferrite media is gyromagnetic and can be characterized by

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a scalar relative permittivity ($\epsilon_f$) and a tensor permeability ($\hat{\mu}$) of the form

$$\hat{\mu} = \mu_0 \hat{\mu}_r = \mu_0 \begin{bmatrix} \mu_{xx} & -j\kappa & 0 \\ j\kappa & \mu_{yy} & 0 \\ 0 & 0 & \mu_{zz} \end{bmatrix} \tag{1}$$

From Maxwell's equations it can be shown that the modes in axially magnetized gyromagnetic media always contain coupled $E_z$ and $H_z$ field components [2] and so a vector analysis is necessary.

The time-harmonic magnetic field within the waveguide satisfies the vector wave equation

$$\nabla \times (\epsilon_f^{-1} \nabla \times \overline{H}) - k_0^2 \hat{\mu}_r \overline{H} = 0 \tag{2}$$

where $k_0$ is the wavenumber of free space. It is assumed here that the waveguide is uniform in the $z$ direction, hence all field components have a $z$-dependence of $e^{-\gamma z}$ where $\gamma$ is the propagation constant. In waveguides containing ferrite the cross-section of the guide may be inhomogeneous with both $\epsilon_f$ and $\hat{\mu}_r$ functions of position. The functional whose stationary point corresponds to the solution of (2) is

$$F(\overline{H}) = \int_S \{\nabla \times \overline{H}^* \cdot \epsilon_f^{-1} \nabla \times \overline{H} - k_0^2 \overline{H}^* \cdot \hat{\mu}_r \overline{H}\} \cdot dS - \int_C \{\overline{H}^* \times \epsilon_f^{-1}(\nabla \times \overline{H})\} \cdot \hat{n}\, dC \tag{3}$$

where $\hat{n}$ is the outward normal around the closed curve $C$ surrounding the waveguide region $S$. This functional was derived by Konrad [5] for applications with anisotropic media which can be described by Hermitian tensors. Regardless of the direction of the applied field, the gyrotropic media of (1) always falls into this category. The waveguide boundary is usually either an electric or a magnetic wall or a combination of both. With any of these boundary conditions the contribution of the line integral term in (3) around the waveguide boundary $C$ is zero, so this term is often ignored in the formulation.

For two-dimensional problems the known $z$-dependence of the fields is substituted into (3), hence there are two unknown parameters associated with each mode: the propagation constant $\gamma = j\beta$ and the free-space wavenumber $k_0$. In this formulation the value of $\beta$ is specified in preference to the wavenumber $k_0$ so that a standard eigenvalue matrix equation is obtained. The usual finite element procedure of approximating the field using a set of locally defined shape functions and a set of unknown field coefficients can be used to discretize the functional in (3), [13]. Ignoring the line integral term in (3) and using (1), the discretized functional gives the following matrix equation

$$F(\overline{H}) = [H_t^* \; -jH_z^*] \begin{bmatrix} A_{tt} & A_{tz} \\ A_{tz}^T & A_{zz} \end{bmatrix} \cdot \begin{bmatrix} H_t \\ jH_z \end{bmatrix} - k_0^2 [H_t^* \; -jH_z^*] \cdot \begin{bmatrix} B_{tt} & 0 \\ 0 & B_{zz} \end{bmatrix} \begin{bmatrix} H_t \\ jH_z \end{bmatrix} \tag{4}$$

where the submatrices are defined as

$$H_t^* A_{tt} H_t = \int_S \{\epsilon_f^{-1} |\nabla_t \times \overline{H}_t|^2 + \beta^2 \epsilon_f^{-1} |\overline{H}_t|^2\}\, dS \tag{5a}$$

$$H_t^* A_{tz} H_z = \int_S \beta \epsilon_f^{-1} \overline{H}_t^* \cdot \nabla_t H_z \, dS \tag{5b}$$

$$H_z^* A_{zz} H_z = \int_S \epsilon_f^{-1} |\nabla_t H_z|^2 \, dS \tag{5c}$$

$$H_t^* B_{tt} H_t = \int_S \{\mu_{xx} |H_x|^2 + \mu_{yy} |H_y|^2 + j\kappa (H_y^* H_x - H_x^* H_y)\}\, dS \tag{5d}$$

$$H_z^* B_{zz} H_z = \int_S \mu_{zz} |H_z|^2 \, dS \tag{5e}$$

Here the known $z$-dependence of the field has been used to give $\nabla \times = \nabla_t \times -j\beta\hat{z}$ and the relative phase difference between the transverse ($H_t$) and axial ($H_z$) magnetic field coefficients is chosen so that the matrix $[A]$ is real symmetric. For ferrites the matrix $[B]$ is Hermitian because of the tensor properties of $\hat{\mu}_r$. The solution is obtained by finding the stationary point of the functional. This corresponds to solving the generalized eigenvalue matrix equation

$$\begin{bmatrix} A_{tt} & A_{tz} \\ A_{tz}^T & A_{zz} \end{bmatrix} \begin{bmatrix} H_t \\ jH_z \end{bmatrix} = k_0^2 \begin{bmatrix} B_{tt} & 0 \\ 0 & B_{zz} \end{bmatrix} \begin{bmatrix} H_t \\ jH_z \end{bmatrix} \tag{6}$$

By setting the phase constant ($\beta$) to zero this formulation becomes ideally suited to evaluating the cut-off planes of gyromagnetic waveguides. The eigenvalue with the lowest value corresponds to the cut-off wavenumber of the dominant mode. The solutions with a cut-off wavenumber of zero were ignored. These solutions correspond to the irrotational fields which are numerical solutions to (6). All eigensolutions with non-zero eigenvalues represent physical modal solutions.

## III. Formulation for the Phase Constant

In order to study the dependence of the phase constant $\beta$ of any particular mode on the direct magnetic field, it is necessary to have a fomulation where the wavenumber $k_0$ is specified and the phase constant is calculated. A simple method for transforming the functional has been described by Cendes and Lee [12]. Their application involved lossless scalar material properties. It can also be used with tensors of the form of (1) which can be split into a transverse and axial part

$$\hat{\mu}_r = \hat{\mu}_{tt} + \mu_{zz}\hat{z} \tag{7}$$

Assuming that $\nabla \times = \nabla_t \times -j\beta\hat{z}$ and splitting the magnetic field into a transverse and an axial part as before, the functional in (3) becomes

$$F(\overline{H}) = \int_S \{\epsilon_f^{-1} |\nabla_t \times \overline{H}_t|^2 - k_0^2 [\overline{H}_t^* \cdot \hat{\mu}_{tt} \overline{H}_t + H_z^* \mu_{zz} H_z] + \epsilon_f^{-1} |\nabla_t H_z + j\beta \overline{H}_t|^2\}\, dS \tag{8}$$

Once more the line integral term is taken to be zero because of the boundary conditions. To obtain a formulation in terms of the phase constant, the field variables can be transformed using

$$\overline{\mathcal{H}}_t = \beta \overline{H}_t \tag{9a}$$

$$\mathcal{H}_z \hat{z} = j H_z \hat{z} \tag{9b}$$

The resulting functional is

$$F(\overline{\mathcal{H}}) = \int_S \{\epsilon_f^{-1}|\nabla_f \times \overline{\mathcal{H}}_t|^2 - k_0^2 \overline{\mathcal{H}}_t^* \cdot \hat{\mu}_{tt}\overline{H}_t + \beta^2[\epsilon_f^{-1}|\nabla_t \mathcal{H}_z + \overline{\mathcal{H}}_t|^2 - k_0^2 \mathcal{H}_z^* \mu_{zz}\mathcal{H}_z]\}\, dS \tag{10}$$

The advantage of this functional is that it gives a generalized eigenvalue matrix problem with the phase constant as the eigenvalue instead of the wavenumber as was used in (3). The discretization of (10) gives a matrix equation

$$F(\overline{H}) = [H_t^* \quad -jH_z^*]\begin{bmatrix} C_{tt} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} H_t \\ jH_z \end{bmatrix} + \beta^2 [H_t^* \quad -jH_z^*] \cdot \begin{bmatrix} D_{tt} & D_{tz} \\ D_{tz}^T & D_{zz} \end{bmatrix} \begin{bmatrix} H_t \\ jH_z \end{bmatrix} \tag{11}$$

where on expansion the following products are defined

$$H_t^* C_{tt} H_t = \int \{\epsilon_f^{-1}|\nabla_t \times \overline{H}_t|^2 - k_0^2 \overline{H}_t^* \cdot \hat{M}_{tt}\overline{H}_t\}\, dS \tag{12a}$$

$$H_t^* D_{tt} H_t = \int \epsilon_f^{-1}|\overline{H}_t|^2\, dS \tag{12b}$$

$$H_t^* D_{tz} H_z = \int \epsilon_f^{-1} \overline{H}_t^* \cdot \nabla_t H_z\, dS \tag{12c}$$

$$H_z^* D_{zz} H_z = \int \epsilon_f^{-1}|\nabla_t H_z|^2 - k_0^2 \mu_{zz}|H_z|^2\, dS \tag{12d}$$

As in (4), the vector $[H_t \quad jH_z]$ represents the unknown magnetic field coefficients. The matrices $[C]$ and $[D]$ are Hermitian and real symmetric respectively. The stationary point of this matrix equation gives

$$\begin{bmatrix} C_{tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} H_t \\ jH_z \end{bmatrix} = -\beta^2 \begin{bmatrix} D_{tt} & D_{tz} \\ D_{tz}^T & D_{zz} \end{bmatrix} \begin{bmatrix} H_t \\ jH_z \end{bmatrix} \tag{13}$$

where $\beta^2$ is the eigenvalue. The largest eigenvalue corresponds to the phase constant of the dominant mode at a specified wavenumber $k_0$.

## IV. Application to Axisymmetric Geometries

In applications which involve longitudinally-magnetized ferrites, circular waveguides are often used. The fundamental geometry is concentric ferrite rods and cylinders in a circular waveguide. This structure has been the subject of a detailed analysis by Waldron [2] and others and its propagation characteristics are well understood. Results for two cases are presented here: the fully-filled case where the radius of the rod $b$ is equal to the radius of the guide $a$: the partially-filled case where $0 < b/a < 1$. In each case the rod has a permeability of the form in (1).

The circular geometry of the structure was modelled using a few convariant-projection elements. Covariant-projection elements are rectangular elements which have a specially designed function space associated with them which prevents the occurrence of spurious mode as described by Crowley *et al.* [8]. These quadratic elements have curvilinear sides so that geometries with curved sides can be modelled easily [14]. In addition the vector shape functions associated with the elements are setup using locally-defined axes. At each node the local axes is defined parallel to the edges of the element. Unlike standard finite elements where all the field components are continuous between elements, only the components tangential to the element edges are made continuous in covariant-projection elements. These elements are well-suited to geometries with material interfaces where both $\epsilon_r$ and $\mu_r$ are discontinuous.

The boundary of the circular waveguide is assumed to be a perfect electric conductor. Thus the required boundary condition is tangential $E$ equal to zero: this boundary condition is the natural one of the formulation in (3). At the waveguide boundary $r = a$, the line integral term (3) becomes an azimuthal $(\theta)$ integral

$$\int_C \{\overline{H} \times \epsilon_f(\nabla \times \overline{H})\} \cdot \hat{n}\, dl = j\omega\epsilon_0 \int_0^{2\pi} \{\overline{H} \times \overline{E}\} \cdot \hat{r}|_{r=a}\, d\theta = j\omega\epsilon_0 \int_0^{2\pi} \{\overline{E} \times \hat{r}\overline{H}\}|_{r=a}\, d\theta = 0 \tag{14}$$

for a perfect conductor.

The axisymmetric geometry of the structure can be used to define planes of symmetry in the field variables. Using these planes of symmetry, the size of the mesh needed is reduced. For any mode, the boundary condition which can be used along a particular azimuthal plane, is identified by examining the tangential magnetic field components along that plane, in this case these will be $H_r$ and $H_z$. In this geometry the modes of interest are hybrid $HE_{\pm1,1}$ and $EH_{0,1}$ modes. In the case of the latter, there is no variation in the azimuthal direction. For the $HE_{\pm1,1}$ mode, both the axial electric and magnetic field components have unity variation in the azimuthal direction and are orthogonal to each other [2]. Note that hybrid mode nomenclature $(HE, EH)$ has been adopted here rather than the limit transverse mode $(TE, TM)$ nomenclature.

With zero magnetization $\kappa = 0$, the tensor permeability is diagonal. From Maxwell's equations, the radial magnetic field component $(H_r)$ is a function of the radial derivative of the axial magnetic field $(\partial H_z/\partial r)$ and the azimuthal derivative of the axial electric field $(\partial E_z/\partial\theta)$ only. The standard Dirichlet and Neumann boundary condition [13] can be used to reduce the mesh to half, quarter or even smaller sections of the waveguide depending upon the mode of interest. Since the magnetic field variable is used in the formulation here a homogeneous Dirichlet boundary conditions corresponds to a magnetic wall and a homogeneous Neumann condition to an electric wall, Hence a homogeneous Neumann boundary condition specified along $\theta = 0$ and $\theta = \pi$ with a half waveguide mesh gives the $EH_{0,1}$ mode and one polarization of the $HE_{\pm1,1}$ mode. The other polarization of the $HE_{\pm1,1}$ mode is obtained by specifying the homogeneous Dirichlet boundary conditions on

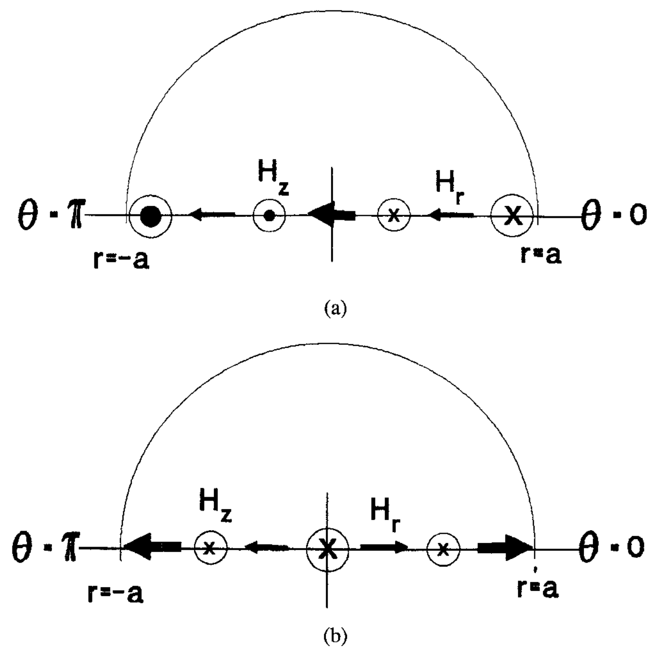


Fig. 1. (a) Antisymmetric magnetic field distribution through a plane of symmetry in an axisymmetric geometry. (b) Symmetric magnetic field distribution through a plane of symmetry in an axisymmetric geometry.

the same half waveguide mesh. These boundary conditions are identical to those used in axisymmetric geometries with scalar media.

When the direct magnetic field is applied $\kappa \neq 0$ the Dirichlet and Neumann boundary conditions can no longer be employed because of the more complicated coupling between the field components introduced by the tensor $\hat{\mu}_r$. From Waldron's analysis the radial magnetic field $H_r$ is a function of the radial and azimuthal derivatives of both axial field components $E_z$ and $H_z$. It is still possible to use the symmetry in the field components to reduce the mesh if periodic boundary conditions are used. For example both polarizations of the $HE_{\pm 1,1}$ mode satisfy

$$\begin{aligned} H_z(r,0) &= \begin{cases} -H_z(r,\pi) & 0 < r \leq a \\ 0 & r = 0 \end{cases} \\ H_r(r,0) &= -H_r(r,\pi) \quad 0 \leq r \leq a \end{aligned} \tag{15}$$

This variation is imposed by specifying an anti-symmetric periodic boundary condition on a half waveguide mesh. For the case of the $EH_{0,1}$ mode the field components satisfy

$$\begin{aligned} H_z(r,0) &= H_z(r,\pi) \quad 0 \leq r \leq a \\ H_r(r,0) &= \begin{cases} H_r(r,\pi) & 0 < r \leq a \\ 0 & r = 0 \end{cases} \end{aligned} \tag{16}$$

This mode is obtained by imposing a symmetric periodic boundary condition on a half waveguide mesh. The field variation associated with (15) and (16) are schematically illustrated in Fig. 1(a) and (b), respectively. The periodic boundary conditions are included in the analysis by constraining the pairs of field variables along the plane of symmetry in the matrix equation. In each case the symmetry of the matrix is not affected. Standard matrix solution subroutines from the NAG library are used to solve the matrix equation in both (6) and (13).

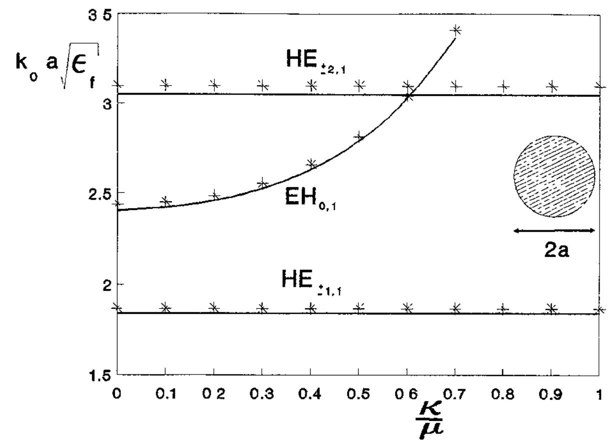


Fig. 2. Analytic (—) and finite element (*) cut-off calculations for the first three modes in axially magnetized ferrite filled circular waveguide. $[\mu_{xx} = \mu_{yy} = \mu \quad \mu_{zz} = 1]$.

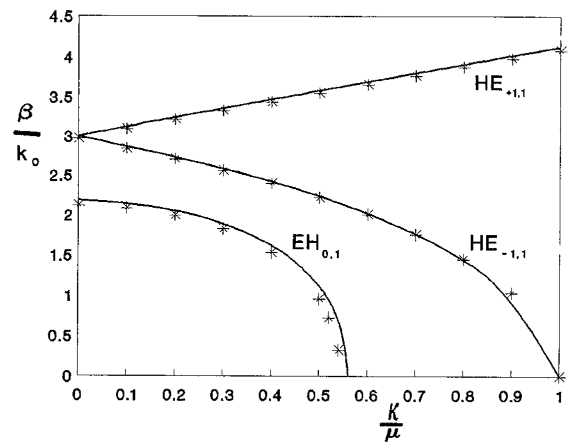


Fig. 3. Analytic (—) and finite element (*) phase constants for the first two modes in axially magnetized ferrite filled circular waveguide. $[k_0 a = 0.750 \quad \epsilon_f = 15 \quad \mu_{xx} = \mu_{yy} = \mu \quad \mu_{zz} = 1]$.

## V. Cut-Off and Phase Constant Calculations

Equations (6) and (13) can be used to examine the cut-off and propagating planes of axially magnetized ferrite waveguides with arbitrary cross-sections. In order to verify these formulations with analytic solutions in the literature, axisymmetric fully-filled and partially-filled circular waveguides have been studied. This further permitted the use of periodic boundary conditions outlined in (15) and (16). All finite element calculations depicted in Figs. 2–5 are compared with published analytical results.

The characteristic equations for the cut-off and propagating planes of fully-filled circular waveguides are well understood [2] and rules of modal hierarchy and nomenclature have previously been enunciated [4]. Calculations at cut-off for the first three modes in a filled waveguide are illustrated in Fig. 2. A mesh of 8 elements modelling half the waveguide was used to obtain these results. Although the dominant $HE_{\pm 1,1}$ mode is independent of magnetization, the first higher order mode $(EH_{0,1})$ has a cut-off number which increases with $(\kappa/\mu)$. Phase constant calculations are depicted in Fig. 3 at a frequency where both the $EH_{0,1}$ and $HE_{\pm 1,1}$ modes are

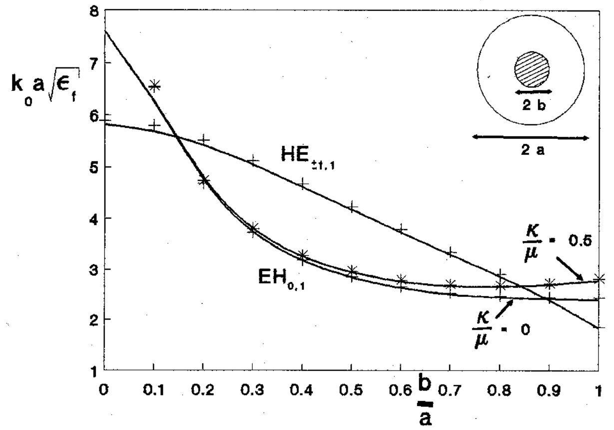


Fig. 4. Analytic (—) and finite element (*) cut-off calculation for the $HE_{\pm 1,1}$ and $EH_{0,1}$ modes in part filled, axially magnetized circular waveguide for two values of $(\kappa/\mu)[\mu_{xx} = \mu_{yy} = \mu \quad \mu_{zz} = 1]$.

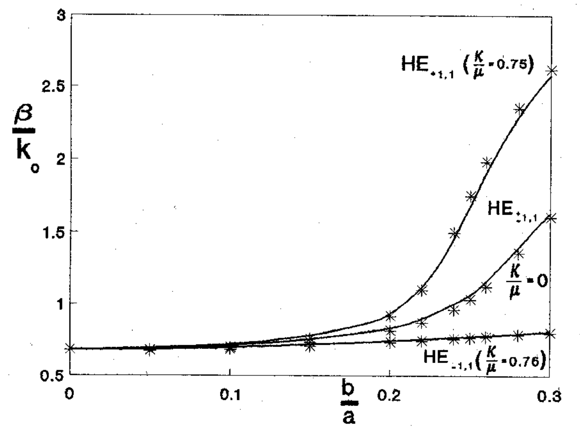


Fig. 5. Waldron's [2] analytic results (—) and finite element (*) phase constant calculations for the dominant mode in axially magnetized partially filled circular waveguide. $[k_0 a = 2.5133 \quad \epsilon_f = 10 \quad \mu_{xx} = \mu_{yy} = \mu \quad \mu_{zz} = 1 \quad \kappa/\mu = 0, 0.75])$.

above cut-off in the unmagnetized fully-filled waveguide. The $EH_{0,1}$ mode cuts-off at the appropriate value of $(\kappa/\mu)$ as predicted with the cut-off plane intersection. Good agreement between numerical and analytical phase constant calculations is shown.

In accordance with the fully-filled waveguide the cut-off numbers of the dominant $HE_{\pm 1,1}$ mode in the partially filled waveguide are invariant with magnetization $(\kappa/\mu)$ whereas the $EH_{0,1}$ mode displays cut-off numbers which are dependent on $(\kappa/\mu)$. Analytic and finite element results for the cut-off plane of the partially-filled case for both these modes are illustrated in Fig. 4 for two values of $(\kappa/\mu)$. Half the waveguide structure was modelled using 12 finite elements. In developing the analytic results Clarricoats' form of the characteristic equation has been used [3]. Note that the modal hierarchy in the inhomogeneous waveguide is dependent on the applied field and the relative rod radius $(b/a)$. For the case illustrated in figure 4 the circularly symmetric $EH_{0,1}$ mode becomes the dominant one over the approximate range $(0.15 < b/a < 0.85)$. In the propagating plane Waldron's results have been used to confirm the accuracy of the finite element formulation for inhomogeneous geometries. Fig. 5 illustrates the phase constants as a function of $b/a$ for the dominant $HE_{\pm 1,1}$ mode for two values of $\kappa/\mu$.

## VI. Conclusions

A three component, vector, finite element formulation which uses covariant-projection elements and avoids spurious modes has been described for axially magnetized ferrite waveguides. In the axisymmetric geometries treated periodic boundary conditions were introduced to improve numerical efficiency. Both cut-off planes and phase constant finite element calculations have been presented and are in good agreement with published analytical results. The formulation discussed may also be applied to longitudinally magnetized plasma waveguides.

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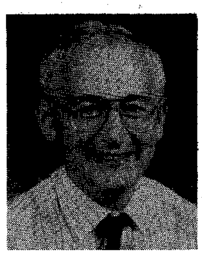


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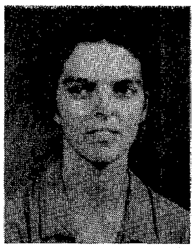

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